KANAZAWA 96-19
August 1996

Recent Results on the Abelian Projection of Lattice Gluodynamics

M.I. Polikarpov[a]

[a]Department of Physics, Kanazawa University, Kanazawa 920-11, Japan
and ITEP, B.Cheremushkinskaya 25, Moscow, 117259 Russia

The abelian projection of lattice gluodynamics is reviewed. The main topics are: abelian and monopole dominance, monopole condensate as the disorder parameter, effective abelian Lagrangian, monopoles in the instanton field, Aharonov – Bohm effect on the lattice.

1. INTRODUCTION

We do not have much intuition in non-abelian theories, whereas our understanding of the Maxwell equations and abelian theories is far better. For this reason people are trying to explain the confinement phenomenon in terms of the abelian projection of nonabelian theory [1]. Different abelian projections lead to different abelian theories, and now it is clear that there exist at least one abelian projection, called the maximal abelian projection (MA) [2], in which the resulting abelian theory is close to the dual Abelian Higgs model, the abelian monopoles are condensed and the linear $q-\bar{q}$ potential can be explained [3] at the level of the classical equations of motion (formation of the dual Abrikosov vortex). The dependence of the abelian theory on the type of the projection is a weak point of this approach, and the popular idea is that all abelian projections can lead to the same physics, if we consider a certain generalized version of the abelian projection, e.g. extended monopoles.

Most of the numerical results are obtained in the MA projection which, for the $SU(2)$ gauge theory, corresponds to the minimization of the functional:

$$R = \int \{[A^1_\mu(x)]^2 + [A^2_\mu(x)]^2\}\, d^4x \,. \qquad (1)$$

The corresponding differential equation is:

$$[\partial_\mu \pm ieA^3_\mu(x)]A^\pm_\mu(x) = 0 \,. \qquad (2)$$

In Section 2 the abelian and monopole dominance is discussed with regard to various abelian projections. In Section 3 we discuss the effective monopole Lagrangian and the phenomenon of the monopole condensation. The relation of monopoles and instantons is reviewed in Section 4. In Section 5 it is shown that an analogue of the Aharonov – Bohm effect can be found in lattice gauge theories.

2. ABELIAN AND MONOPOLE DOMINANCE

2.1. Maximal Abelian Projection

The notion of the "abelian dominance" introduced in [4] means that the expectation value of the physical quantity $<\mathcal{X}>$ in nonabelian theory coincides with the corresponding expectation value in the abelian theory obtained by the abelian projection. The monopole dominance means that the same quantity can be calculated in terms of the monopole currents extracted from the abelian fields. If we have N configurations of the nonabelian fields on the lattice, the abelian dominance means that:

$$\frac{1}{N}\sum_{conf}\mathcal{X}(\hat{U}_{nonabelian}) =$$
$$\frac{1}{N}\sum_{conf}\mathcal{X}'(U_{abelian}) = \frac{1}{N}\sum_{conf}\mathcal{X}''(j)\,. \qquad (3)$$

Here each sum is taken over all configurations; $U_{abelian} = e^{i\theta_l}$ is the abelian part of the nonabelian field $\hat{U}_{nonabelian}$, j is the monopole current extracted from $U_{abelian}$. It is clear that $\frac{1}{N}\sum_{conf}\mathcal{X}(\hat{U}_{nonabelian})$ is a gauge invariant

quantity, while the abelian and the monopole contributions depend on the type of the abelian projection. In numerical calculations the equalities (3) can only be satisfied approximately.

Among the well-studied problems is that of the abelian and the monopole dominance for the string tension [4–7]. In this case, $\mathcal{X}(\hat{U}_{nonabelian}) = \sigma_{SU(2)}$, $\mathcal{X}(U_{abelian}) = \sigma_{U(1)}$ and the string tension $\sigma_{SU(2)}$ ($\sigma_{U(1)}$) is calculated by means of the nonabelian (abelian) Wilson loops, $Tr\prod_{l\in C}\hat{U}_l$ ($\prod_{l\in C}e^{i\theta_l}$). An accurate numerical study of the MA projection of $SU(2)$ gluodynamics on 32^4 lattice at $\beta = 2.5115$ is performed in ref.[7]. The abelian and the nonabelian potentials are shown in Fig. 1 (taken from [7]). The contribution of the photon and the monopole parts to the abelian potential is shown in Fig. 2 [7].

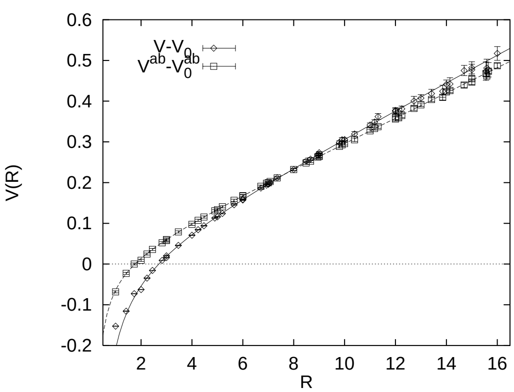

Figure 1. Abelian and nonabelian potentials

The differences in the slopes of the linear part of the potentials in Fig. 1 and Fig. 2 yield the following relations: $\sigma_{U(1)} \approx 92\%\,\sigma_{SU(2)}$, $\sigma_j \approx 95\%\,\sigma_{U(1)}$, where σ_j is the monopole current contribution to the string tension. It is important to study a widely discussed idea that in the continuum limit ($\beta \to \infty$) the abelian and the monopole dominance is exact (3): $\sigma_{SU(2)} = \sigma_{U(1)} = \sigma_j$.

There are many examples of the abelian and the monopole dominance in the MA projection. The monopole dominance for the string tension has been found in the $SU(2)$ positive plaquette model in which Z_2 monopoles are suppressed [8], it has also been found for the $SU(2)$ string tension at finite temperature [9] and for the string

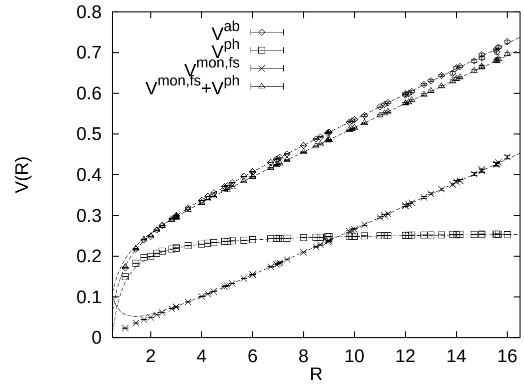

Figure 2. The abelian potential (diamonds) in comparison with the photon contribution (squares), the monopole contribution (crosses) and the sum of these two parts (triangles).

tension in the $SU(3)$ gluodynamics [10]. The abelian and the monopole dominance for the $SU(2)$ gluodynamics has been found in [11,12] for the Polyakov line and for the critical exponents for the Polyakov line, for the value of the quark condensate, for the topological susceptibility and also for the hadron masses in the quenched $SU(3)$ QCD with Wilson fermions [13].

2.2. Two Open Problems

Consider the adjoint sources in the $SU(2)$ gluodynamics. After the abelian projection, the corresponding Wilson loop can be represented as a sum of the charge two, W_2, and the charge zero, W_0, contributions. This fact give rise to a paradox discussed in refs. [14–17]: for the zero $U(1)$ charge there is no confinement (therefore there is no area low for the sum $W_2 + W_0$), but for the $SU(2)$ gluodynamics at the intermediate distances there exists string tension for the adjoint sources ("Casimir scaling" [14]). A partial solution of this problem has been found in refs. [16,17]: it is possible to reproduce the $SU(2)$ string tension for the adjoint sources using only abelian variables (see Fig.11 of ref.[16] and Fig.3 of ref.[17]).

Another widely discussed question is that, possibly, the property of the abelian dominance can be proved analytically, and is, therefore, trivial. The idea is as follows. Consider an irreducible correlator $G(x) = < A(x)A(0) >$. By the spectral theorem $G(x) \approx Ce^{-m|x|}$ at large $|x|$; the con-

stant C depends on the choice of A, but the mass m is the same for all A with the same quantum numbers. Similar arguments based on the transfer matrix approach show that the string tension should be the same for the $SU(2)$ sources in the full $SU(2)$ theory and for the $U(1)$ sources after the abelian projection. But for the MA projection the resulting $U(1)$ theory is nonlocal in space and in time and the use of the transfer matrix approach and/or spectral theorems is questionable.

2.3. Various Abelian Projections

Different abelian projections lead to different $U(1)$ gauge theories. The "extreme" example is the "minimal abelian projection" [18], in which the properties of the monopole currents differ much from those in the MA projection. The projection in which the field strength $\hat{F}_{12}$ is diagonalized also yields the results which are different from those in the MA projection [19–21]. The projection, corresponding to the diagonalization of the Polyakov line is closer to the MA projection, but still the results are not the same [20,22,21]. For instance, the Abrikosov vortices are suppressed in this projection compared to the MA projection [22]. There are two new examples of the abelian projection (mAA and mAMD) [23] with the results very close to the MA projection: the abelian and the monopole dominance is found in these projections (see Fig. 5 of ref.[23]). Due to the presence of the Gribov copies it is difficult to fix numerically the MA projection, just as the projections suggested in ref.[23]. A smooth abelian projection which is free from the Gribov copies is suggested in ref.[24], a numerical study shows that the properties of the monopole currents in this projection are close to the properties of the monopole currents in the MA projection; the abelian dominance has not yet been studied.

In ref.[15] the "Maximal Z_2 gauge" is suggested in which the $SU(2)$ fields are projected on the Z_2 fields. It is instructive that the approximate abelian dominance (center dominance) for the string tension exists in this projection. This fact allows one to discuss [15] the "spaghetti vacuum model" as the supplementary model to the model of the dual superconducting vacuum. Another observation from ref.[15] should be mentioned: if we perform the MA projection only for the gauge fields $U_{x,x+\hat{\mu}}$ with $\mu = 1,2$ and do not fix the MA projection for $\mu = 3,4$, then the abelian dominance takes place for the string tension constructed from Wilson loops in $1-2$ plane.

3. MONOPOLE CONDENSATE AND EFFECTIVE MONOPOLE ACTION

3.1. Disorder Parameter for the Deconfinement Phase Transition

If the vacuum of the $SU(2)$ gluodynamics in the abelian projection is similar to the dual superconductor, then the value of the monopole condensate should depend on the temperature as a disorder parameter: at low temperatures it should be nonzero, and it should vanish above the deconfinement phase transition. A numerical study of the monopole condensate has been recently performed by three teams [20,25–27].

The logarithmic derivative of the monopole creation operator with respect to β ($\rho = \partial\varphi/\partial\beta$) for $SU(2)$ lattice gluodynamics is studied in refs.[20,25], where it is shown to have a peak just at the point of the phase transition. A similar operator exhibits the same behavior in the lattice compact electrodynamics, in the $SU(3)$ lattice gauge theory, and in the $3D$ XY model [25,28].

Another form of the monopole creation operator is studied in ref.[26]. This form is similar to that suggested by Fröhlich and Marchetti [29] for the compact electrodynamics. For the $SU(2)$ lattice gluodynamics in the MA projection, it is convenient to study the probability distribution of the value of the monopole creation operator (similar calculations were performed for the compact electrodynamics in ref. [30]). It occurs that at low temperatures, below the deconfinement phase transition the maximum of the distribution is shifted from zero, which means that the effective constraint potential is of the Higgs type. Above the phase transition the minimum of the potential, φ_C, (the maximum of the monopole field distribution) is at the zero value of the monopole field. The dependence of the quantity φ_C (which is proportional to the value of the monopole condensate) on β is shown in Fig. 3. It is obvious that φ_C behaves as the dis-

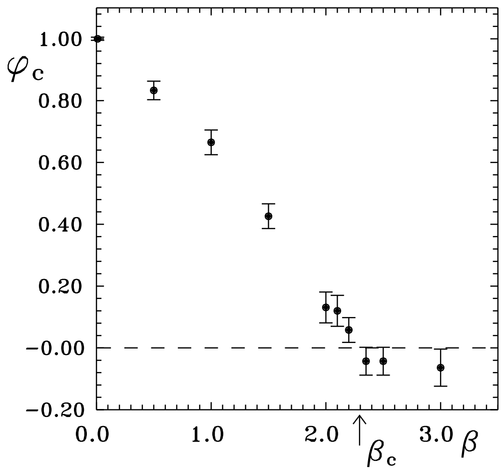

Figure 3. The position of the minimum of the effective constraint potential for the monopole creation operator.

order parameter. To get this result the calculations are performed on the lattices of the size 4×8^3, 4×10^3, 4×12^3, 4×14^3, 4×16^3 and the data for φ_C are extrapolated to the infinite volume.

The monopole creation operator [31] in the monopole current representation is studied in ref.[27]. First the monopole action is reconstructed from the monopole currents in the MA projection, and after that the expectation value of the monopole creation operator is calculated in the quantum theory of monopole currents. Again, the monopole creation operator depends on the temperature as the disorder parameter.

3.2. Effective Monopole Action

The examples discussed in Sect. 3.1 show that there exists the monopole condensate in the confinement phase of the lattice gluodynamics. Thus the simplest (i.e., with the minimal number of derivatives) effective Lagrangian for the abelian fields (diagonal gluon fields) should be equivalent to the Lagrangian of the dual Abelian Higgs model. In this model, the confinement of quarks exists at the classical level. It is important to find out whether the effective Lagrangian for the abelian field (diagonal gluon field) in the continuum limit ($\beta \to \infty$) is close to the Lagrangian of the Abelian Higgs model. The effective Lagrangian for the monopole currents can be more easily reconstructed from the numerical data [23]. For the $SU(2)$ lattice gauge theory in the MA projection the coefficients of the Lagrangian for the extended monopoles [19] seem to scale [23], which means that there exists a continuum limit of the effective action. Some preliminary results have been obtained by a similar study of the monopole Lagrangian for the lattice $SU(3)$ gluodynamics [23].

4. MONOPOLES AND INSTANTONS

Since monopoles are responsible for the confinement, it is important to find a general class of nonabelian fields which generate monopoles, in particular, in the MA projection. This is a rather complicated problem. There exists the projection independent definition of the monopole [32]. Still it is unclear how these monopoles are related to the abelian monopoles and what is the confinement mechanism if monopoles are nonabelian. As claimed in [33], there are some classical structures (bumps in the field strength) which are correlated with the string tension. It is unclear wether these structures are related to monopoles.

At present, there exists only one carefully studied example: it occurs that instantons and monopoles in the MA projection are interrelated. The simplest solution [34] of the problem, shown in Fig. 4, consists of the straight line monopole trajectory which goes through the center of the instanton.

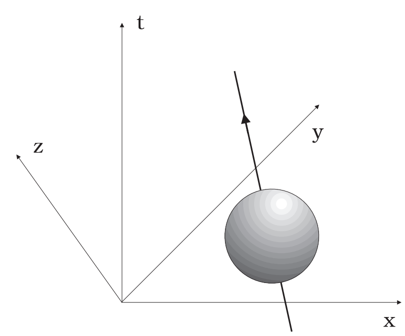

Figure 4. Mononopole current in the instanton field. Sphere represents an instanton, line is the monopole current.

A more complicated solution [35], shown in Fig. 5, consists of the circular monopole current of radius R, and the instanton of the width ρ at the center of the monopole trajectory.

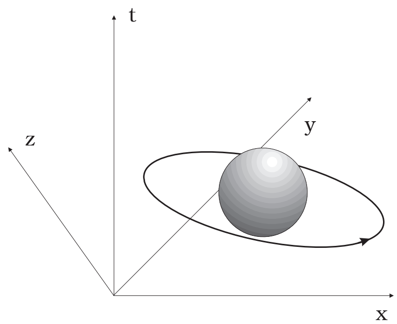

Figure 5. Another solution of eq. (2), the notations are the same as in Fig. 4.

It occurs that both solutions satisfy equation (2), but the minimization condition of (1) holds only for the second solution (Fig. 5) in the limit $R \to 0$. It is important that the leading correction to the minimization condition of (1) is of the order of $(R/\rho)^4 \ln(R/\rho)$ [35]. Therefore any small quantum fluctuations (or even the coarseness of the lattice) can create small monopole loops.

The solutions, shown in Figs. 4,5 are observed for the $SU(2)$ instantons in the MA projection on the lattice [36–38]. Moreover, there exists a correlation between the topological charge and the monopole currents for the cooled and non–cooled lattice field configurations in the MA projection [39,40]. The relation observed between instantons and monopoles allows one to discuss instanton–monopole models of the QCD vacuum [41].

In the MA projection the nonabelian part of the gluon field is suppressed and, therefore, the abelian field corresponding to the instanton field is almost selfdual and the magnetic current $j_\mu = \frac{1}{2}\partial_\nu \varepsilon_{\mu\nu\alpha\beta} f_{\alpha\beta}$ should be accompanied by the electric current $j_\mu = \partial_\nu f_{\mu\nu}$. This effect – the dyon creation by the instanton field – is observed for the instantons on the lattice [38]. If dyons (not monopoles) are condensed, then the Abrikosov–Nielsen–Olesen string dynamics may be very nontrivial and the QCD strings may be fermionic (E. Akhmedov, M. Chernodub and M. Polikarpov, in preparation).

5. AHARONOV – BOHM EFFECT ON THE LATTICE

There is a field theoretical analogue [42] of the Aharonov-Bohm effect. The simplest example is the Abelian Higgs theory. It is possible to represent the partition function of this theory as a sum over closed surfaces [43], which are the world sheets of the Nielsen–Olesen strings: $\mathcal{Z} = \sum_\sigma \exp\{-S(\sigma)\}$. In this representation the expectation value of the Wilson loop for the charge M is:

$$< W_M(\mathcal{C}) >= \frac{1}{\mathcal{Z}} \sum_\sigma \exp\{-S_{local}(\sigma,\mathcal{C}) + 2\pi i \frac{M}{N} \mathbb{L}(\sigma,\mathcal{C})\} \qquad (4)$$

Here N is the charge of the Higgs field. The long–range interaction described by the term which is proportional to the linking number $\mathbb{L}$ of the string world sheet σ and the world line $\mathcal{C}$ [1] of the test charge is a four–dimensional analogue of the Aharonov–Bohm effect: strings correspond to solenoids which scatter charged particles.

This topological interaction was found numerically [44] in the $3D$ Abelian Higgs model. In the abelian projection of gluodynamics the off–diagonal gluons carry charge 2, the test quark in the fundamental representation has charge 1, and the situation is quite similar to the Abelian Higgs model. The problem is how to construct the scalar field (an analogue of the Higgs field) from the vector charged field (off–diagonal gluon). This can be done in several ways. There are indications that for a new type of the abelian projection of the $SU(2)$ lattice gluodynamics there exists topological interaction (M.N. Chernodub and M.I. Polikarpov, work in progress). It occurs that

$$< AB > - < A > \cdot < B > \neq 0, \qquad (5)$$

[1] In three dimensions there is the linking of closed curves, the simplest example is shown in Fig. 6. In four dimensions there exists the linking of a closed surface and a closed curve, see Fig. 7.

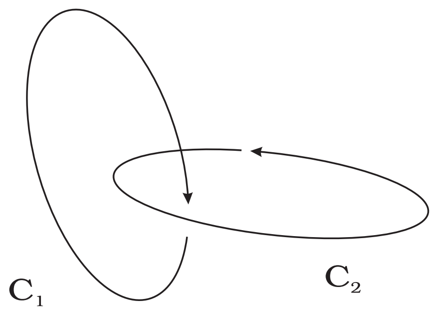

Figure 6. The linking of two curves $\mathcal{C}_1$ and $\mathcal{C}_2$ in three dimensions.

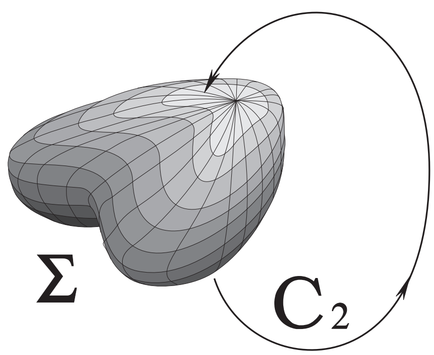

Figure 7. The linking of the curve $\mathcal{C}_2$ and the closed surface Σ in four dimensions.

where $A = W(\mathcal{C})$ is the Wilson loop, and $B = \exp\{\pi i \mathbb{L}(\sigma, \mathcal{C})\}$. The details will be given in a separate publication.

6. CONCLUSIONS AND ACKNOWLEDGMENTS

The facts described in this talk show that there are many physical effects in the abelian projection of gluodynamics. It occurs that in the MA projection (almost) all physical information is shifted to the abelian part of the gluon field. Probably this is a nontrivial fact, which is due to some small dynamical parameter.

I am very obliged to many people who supplied me with data and helped me to prepare this small review. My special thanks are to V. Bornyakov, R. Brower, A. Di Giacomo, R. Haymaker, H. Markum, G. Poulis, A. van der Sijs and T. Suzuki. The help of my collaborators M. Chernodub, F. Gubarev, A. Veselov and E. Akhmedov was very important. This work is supported by the JSPS Program for Japan – FSU scientists collaboration, by the Grant INTAS-94-0840, and by the Grant No. 96-02-17230a, financed by the Russian Foundation for Fundamental Sciences.

REFERENCES

1. G. 't Hooft, Nucl. Phys., B190 [FS3] (1981) 455.
2. A.S. Kronfeld, M.L. Laursen, G. Schierholz and U.J. Wiese, Phys.Lett.198B (1987) 516; A.S. Kronfeld, G. Schierholz and U.J. Wiese, Nucl.Phys. B293 (1987) 461.
3. S. Mandelstam, Phys. Rep., 23C (1976) 245; G. 't Hooft, "High Energy Physics", Zichichi, Editrice Compositori, Bolognia, 1976.
4. T. Suzuki and I. Yotsuyanagi, Phys. Rev., D42 (1990) 4257.
5. S. Hioki et. al.,Phys.Lett., B272 (1991) 326; ERRATUM-ibid. B281 (1992) 416.
6. J.D. Stack, S.D. Neiman and R.J. Wensley, Phys.Rev., D50 (1994) 3399.
7. G.S. Bali, V. Bornyakov, M. Mueller-Preussker, K. Schilling; hep-lat/9603012, to be published in Phys.Rev.D.
8. J.D. Stack, S.D. Neiman, Phys.Lett., B377 (1996) 113.
9. J.D. Stack, R.J. Wensley and S.D. Neiman, preprint OUTP-96-26-P, May 1996; hep-lat/9605045;
   J.D. Stack, these proceedings; hep-lat/9607064.
10. K. Yee, Nucl. Phys. B (Proc. Suppl.), 34 (1994) 189.
11. S. Kitahara, preprint KANAZAWA-96-12, these proceedings.
12. S. Ejiri, these proceedings; hep-lat/9608001.
13. T. Suzuki, S. Kitahara, T. Okude, F. Shoji, K. Moroda and O. Miyamura, Nucl. Phys. B (Proc. Suppl.) 47 (1996) 374, hep-

lat/9509016;
T. Suzuki, preprint KANAZAWA-96-08, Jun 1996, hep-lat/9606015.
14. L. Del Debbio, M. Faber, J. Greensite and S. Olejnik, Phys.Rev., D53 (1996) 5891.
15. L. Del Debbio, M. Faber, J. Greensite and S. Olejnik, these proceedings; hep-lat/9607053.
16. G. Poulis, preprint NIKHEF-95-064, Jan 1996.; hep-lat/9601013.
17. G. Poulis, these proceedings, hep-lat/9608054.
18. M.N. Chernodub, M.I. Polikarpov and A.I. Veselov, Phys. Lett. B342 (1995) 303.
19. T.L. Ivanenko, A.V. Pochinskii and M.I. Polikarpov, Phys.Lett. B252 (1990) 631.
20. L. Del Debbio, A. Di Giacomo, G. Paffuti and P. Pieri, Phys.Lett. B355 (1995) 255.
21. A. Hart and M. Teper, preprint OUTP-96-36-P; hep-lat/9606022.
22. K. Bernstein, G. Di Cecio and R.W. Haymaker, preprint LSUHE-213-1996, Jun 1996, hep-lat/9606018;
K. Bernstein, G. Di Cecio and R.W. Haymaker, these proceedings; hep-lat/9607075.
23. T. Suzuki et.al., these proceedings; hep-lat/9607054.
24. A.J. van der Sijs, these proceedings; hep-lat/9608041.
25. A. Di Giacomo, preprint IFUP-TH-31-96; hep-lat/9606001.
26. M.N. Chernodub, M.I. Polikarpov and A.I. Veselov, Nuclear Physics B (Proc. Suppl.) 49 (1996) 307; hep-lat/9512030.
27. N. Nakamura, V. Bornyakov, S. Ejiri, S. Kitahara, Y. Matsubara and T. Suzuki, these proceedings; hep-lat/9608004.
28. G. Di Cecio, A. Di Giacomo, G. Paffuti and M. Trigiante, preprint IFUP-TH-13-96, Mar 1996, cond-mat/9603139;
G. Di Cecio, A. Di Giacomo, G. Paffuti and M. Trigiante, these proceedings; hep-lat/9608014.
29. J. Fröhlich and P.A. Marchetti, Europhys. Lett. 2 (1986) 933.
30. M.I. Polikarpov, L. Polley and U.J. Wiese, Phys.Lett.B253 (1991) 212.
31. T. Kennedy and C. King, Comm.Math.Phys., 104 (1986) 327.
32. M. Zach, M. Faber and P. Skala, these proceedings; hep-lat/9608009.
33. A. Gonzalez-Arroyo and A. Montero, preprint FTUAM/96-13, hep-th/9604017;
A. Gonzalez-Arroyo and A. Montero, these proceedings, hep-lat/9608035.
34. M.N. Chernodub and F.V. Gubarev, JETP Lett.62 (1995) 100; hep-th/9506026.
35. R.C. Brower, K.N. Orginos and C.I. Tan, these proceedings; hep-lat/9608012.
36. A. Hart and M. Teper, preprint OUTP-95-44-P; hep-lat/9511016.
37. A. Gonzalez-Arroyo and M.I. Polikarpov, unpublished.
38. V. Bornyakov and G. Schierholz, preprint DESY 96-069; hep-lat/9605019.
39. H. Suganuma, A. Tanaka, S. Sasaki and O. Miyamura, in Proceedings of the LATTICE 95 symposium, Nucl. Phys. B (Proc. Suppl.) 47 (1996); hep-lat/9512024.
40. M. Feurstein, H. Markum and St. Thurner, these proceedings, hep-lat/960803;
M. Feurstein, H. Markum and St. Thurner, these proceedings, hep-lat/9608038.
41. M. Fukushima, S. Sasaki, H. Suganuma, A. Tanaka, H. Toki and D. Diakonov, hep-lat/9608084.
42. M.G. Alford and F.Wilczek, Phys. Rev. Lett., 62 (1989) 1071;
M.G. Alford, J. March–Russel and F.Wilczek, Nucl. Phys., B337 (1990) 695;
J. Preskill and L.M. Krauss, Nucl. Phys., B341 (1990) 50;
M. Alford, K-M. Lee, J. March-Russel and J. Preskill, Nucl. Phys., B384 (1992) 251.
43. M.I. Polikarpov, U.J. Wiese and M.A. Zubkov, Phys.Lett. B309 (1993) 133.
E.T. Akhmedov, M.N. Chernodub, M.I. Polikarpov and M.A. Zubkov, Phys.Rev. D53 (1996) 2087-2095.
44. M.N. Chernodub, F.V. Gubarev and M.I. Polikarpov, preprint ITEP-TH-29-96, May 1996, hep-lat/9607045;
M.N. Chernodub, F.V. Gubarev and M.I. Polikarpov, these proceedings; hep-lat/9608075.